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Power spectral depth from residual magnetic field of Calabar flank, southeastern, Nigeria.

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ABSTRACT

Fourier transformation of aeromagnetic anomaly data of Calabar Flank from space domain to frequency domain yielded a method for source depth determination. From the analysis of the magnetic data, progressively deeper sources correspond to the lower frequency components while the shallower sources are indicated by higher frequency components. The power spectrum analysis gave a two-depth source model with the deeper sources (intra basement sources) indicating a depth of 5.8km while the shallower sources (sedimentary section) gave a value of 3.11km. These source depths are compatible with earlier results obtained from gravity surveys within and adjacent to the flank.

INTRODUCTION

The application of spectral analysis to magnetic data interpretation is based on the fact that sharp anomalies with rapid decay of amplitude away from the centre of the anomaly will be characterized by high frequency content while a broad anomaly with a slow decay in amplitude will have its spectra confined in the low frequency end. That is, the two types of anomalies (broad and sharp) from deep and shallow sources respectively will differ considerably in their spectral characteristics. This implies that a magnetized body very close to the surface of the earth will produce a much sharper anomaly than the body located deeper in the earth. The most popular method for depth of magnetic source determination is based on the method of spector and Grant (1970). This technique shows that the characteristics of the residual magnetic field can be analyzed by transforming the magnetic data from the space domain to the frequency domain and then study their frequency characteristics. Spector and Grant (1970) and Hahn et al. (1976) showed that spectral analysis is very effective for depth estimate of layered structures. Using the frequency domain approach they concluded that the power or energy spectrum of the anomaly will have dominant high frequency component when the anomaly is continued to the proximity of the source. The shallower sources will thus give flatter power spectrum and the deeper sources will give steeper power spectrum. Bhattacharyya (1966a) showed that the continuous or discrete spectrum from which the power spectrum can be calculated can be used to estimate the depth of the magnetic body. This is based on Fourier series which consists of different frequencies that combine together to form a potential field data. The amplitude and phase relationships among these frequencies constitute the complex line spectrum.

The economic potential of the Calabar Flank in terms of solid and fuel minerals has remained a topical issue. Serious exploration activities (seismic data acquisition) have been confined to the adjacent Niger Delta basin. There are indications that some detailed exploration activities may commence in the Flank to tap the mineral resources judging from the Federal government's policy to sustain and diversify the economy. The determination of sediment thickness and depth to intrabasement sources are invaluable information to the initial step in the assessment of the economic potential of the Flank. Earlier gravity survey of part of the Flank (Fairhead and Okereke, 1987; Fairhead *et al*; 1991) suggested sediment thickness of 3000m. Due to non-uniqness of potential field results, the power spectra depth approach of the magnetic data provides an alternative method to determine the thickness of the sedimentary section and the depth to intrabasement features particularly over areas where ground surveys were not dense.

THEORY AND METHODOLOGY

Theoretical background

A periodic function $f(x)$ of the independent variable, x the dimension of which is length, may be represented as a Fourier series:

$$f(x)=\frac{a_0}{r}+\sum_{n=1}^{\infty}[a_n\cos(n\omega x)+b_n\sin(n\omega x)] \qquad (1)$$

Where ω is the angular frequency in radians per unit length $(\omega=\frac{2\Pi}{x})$.

Kangkolo (1995) showed that Equation (1) may be expressed in exponential form. Thus,

$$f(x)=\sum_{-\infty}^{\infty}f(n)e^{jn\omega x} \qquad (2)$$

$$f(n)=\frac{1}{x}\int_{-x/2}^{x/2}f(x)e^{-jnwx}dx \qquad (3)$$

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where $f(n)$ is the complex line spectrum of $f(x)$.When the period is allowed to tend to infinity, the function $f(x)$ begins to approach a periodic function or a transient function which contain all possible frequencies. In such a case the summation sign in the series in (2) is replaced by an integral and equation (2) reduces to:

$$f(x)=\frac{1}{2\Pi}\int_{-\infty}^{\infty} e^{j\omega x}dx \quad (4)$$

The reciprocal relations for $f(x)$ in equation (4) may be written as

$$f(\omega)=\int_{-\infty}^{\infty} f(x)e^{-j\omega x}dx \quad (5)$$

$$f(x)=\frac{1}{2\Pi}\int_{-\infty}^{\infty} f(\omega)e^{j\omega x}d\omega \quad (6)$$

It is evident from equation (6) that a periodic function $f(x)$ may be thought of as a synthesis of an infinite aggregate of sinusoids $e^{j\omega x}$ of all angular frequencies ω in the range $(-\infty,\infty)$, having a complex continuous spectrum of a periodic function $f(x)$.

Bhattacharyya [1966(b)], shows that when the theory of Fourier Transformation is extended to the two-dimensional case, the transform pair (5) and (6) may be written as

$$G(U,V)=\int_{-\infty}^{\infty} g(x,y)e^{-j(ux+vy)}dxdy$$

$$g(x,y)=\frac{1}{4\Pi}\int_{-\infty}^{\infty} G(U,V)e^{j(ux+vy)}dudv \quad (7)$$

where u and v are the angular frequencies in x and y directions respectively.

Generally, $G(u, v)$ is complex and contains information on the amplitude and phase relationships about all the frequencies that make up the two dimensional function $g(x,y)$. If $G(u,v)$ is broken into its real and imaginary components:

$$G(u,v)=p(u,v)+jQ(u,v) \quad (8)$$

The amplitude spectrum of $G(u,v)$ is

$$A(u,v)=|G(u,v)|=\left[p^2+Q^2\right]^{1/2} \quad (9)$$

Its phase density spectrum is

$$\theta(u,v)=\tan^{-1}\left(\frac{Q}{P}\right) \quad (10)$$

The energy spectrum is

$$E(u,v)=|G(u,v)|^2=(p^2+Q^2) \quad (11)$$

Oppenheim and Schafer (1975) suggested that for the treatment of sampled data, the discrete form of $G(u,v)$ has to be explored. Thus, the discrete Fourier Transform pair for the two-dimensional data sequence $g(m,n)$ is given by:

$$G(K,I)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} g(m,n)W_M{}^{Km}W_N{}^{Kn} \quad (12)$$

$$g(m,n)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} G(K,I)W_M{}^{-km}W_N{}^{-kn} \quad (13)$$

where, $W_M=e^{-j2\Pi/M}$ (14)

$$W_N=e^{-j2\Pi/N} \quad (15)$$

Power spectrum and depth

For the purpose of analyzing aeromagnetic maps, Spector and Grant (1970) assumed the ground to consist of number of independent ensembles of rectangular, vertical sided parallelpipeds. Each ensemble is characterized by a joint frequency distribution for the depth, h width d, length b, depth extent t. To consider the power spectrum of the total magnetic field intensity anomaly over a single rectangular block, the expression which was first given by Bhattacharyya (1966a) is transcribed into polar wave number coordinates in the u,v frequency plane (Kangkolo, 1995). Thus if

$$r=(u^2+v^2)^{1/2} and \theta=\tan^{-1} ut$$

$$=4\Pi^2k^2e^{2hr}(1-e^{-tr})^2 s^2(r,\theta)R_T{}^2(\theta)R_K(\theta) \quad (16)$$

$\frac{K}{4ab}$ is the magnetic moment/unit volume of the body (k is a magnetic moment/unit depth).

$$S(r,\theta)=\frac{\sin(\cos^{-1}\theta)}{\cos^{-1}\theta}\cdot\frac{\sin(br\cos\theta)}{br\cos\theta}$$

$$R_T{}^2(\theta)=[n^2+(I\cos\theta+m\sin\theta)^2]$$

$$R_T{}^2(\theta)=[N^2+(L\cos\theta+M\sin\theta)^2]$$

I,m,n are direction cosines of the geomagnetic field vectors and L,M,N are direction cosines of the magnetic moment vectors.

In order to define a new form for the power spectrum, spector and Grant (1970) made further assumptions. They opined that for a moderately large number of bodies the average values of inclination and declination of the magnetic vector will not differ significantly from the inclination and declination of the geomagnetic field. Accordingly, they obtained the expression for the power spectrum as

$$E(r,\theta)=4\Pi^2K^2[(e^{-2hr})(1-e^{-hr})^2S^2(r,\theta) \qquad (17)$$

The $\exp(-2hr)$ term is the overriding factor in the power spectrum.

If there are two sets of sources, they can be identified by the sharp change in the spectral decay rate. The power spectrum of the double ensemble will consists of two parts. The first will relate to the deeper source which is relatively strong at the low frequencies and decay away rapidly. The second arises from the shallower ensemble of sources and dominates the high frequency end of the spectrum. Generally, the radial spectrum may be conveniently approximated by straight line segments, the slopes of which relate to the depths of the possible layers (Hahn *et al*, 1976).

Application to residual magnetic field data of Calabar Flank

The Calabar Flank is a sedimentary basin extending from the southern margins of the igneous Oban Massif to the hinge line of the Niger Delta basin (fig.1).The Flank extends to the Cameroun volcanic ridge in the east. Petters(1980), opined that the Calabar Flank is southeast extention of the Benue aulacogen. Northwest – southeast trending basement structures underlie the Flank [Reijers and Petters, 1997] and define the Ituk high and the Ikang trough; thus relating the Calabar Flank to the south Atlantic Cretaceous marginal basins with similar horst and graben structures.

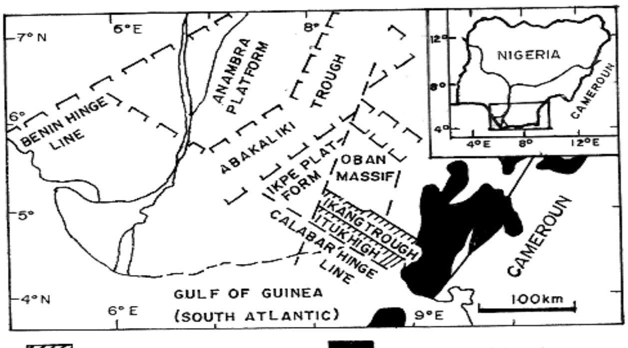

Fig.1. Location map and structural elements of the Calabar Flank and adjacent areas (After Nyong and Ramanathan, 1995).

The stratigraphic succession compiled by Petters (1982) shows that the Flank is mostly of Cretaceous age, comprising a basal Neocomiam- Aptian syn-rift fluvial sandstone, the Awi Formation and the marine post – rift Odukpani Group. The Odukpani Group consists of the middle Albian Mfamosing limestone,the Late Albian Ekenkpon shale and the Coniacian New Netim marl. It is unconformably covered by the Nkporo shale. Tertiary marine shale and regressive sandstone overlie the Cretaceous succession. Reijers and Petters(1997) opined that the total sediment thickness is over 3500m. Nyong (1995) opined that after the initial rifting episode in Calabar Flank, the area underwent a different tectonic and stratigraphic development compared to the adjacent Anambra and Southern Benue Trough sedimentary basins. The initial rifting of the southern Nigerian margin produced two principal sets of faults, a NE-SW and NW-SE system. The former set of faults bound the Benue depression while the later sets were more prominent and active in Calabar Flank. Two major tectonic elements in the Calabar Flank include the Ikang Trough which for most depositional history was a mobile depression and the Ituk high that was a stable to somewhat mobile submarine ridge.

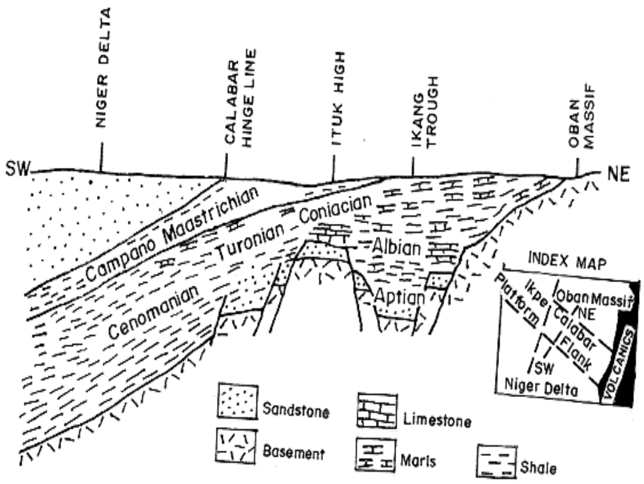

Fig.2.Structural elements and conceptual subsurface distribution of Cretaceous sediments in the Calabar Flank (After Nyong, 1995).

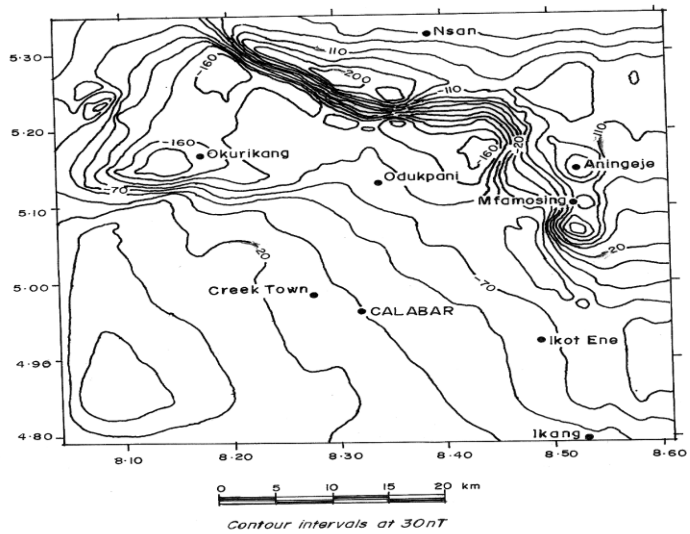

Fig.3. Residual magnetic field of Calabar Flank characterized by positive and negative magnetic anomalies.

These structural styles (Fig.2) are reflected in the sedimentation facies distribution in the area (Peters, 1982). The structural alignment of the Calabar Flank is in NW-SE direction.

The two-dimensional Fourier Transform can be obtained from equation (12) using the digitized residual values, ΔT (x .y) from fig.3

consisting of M rows and N columns in the x-y plane. The evaluation is done using an algorithm that is a two-dimensional extension of the Fast Fourier transform (Oppenheim and Schaffer, 1975). The frequency intervals are subdivided into sub-intervals which lie within one unit of frequency range. The average spectrum of all the partial waves falling within this frequency range is calculated and the resulting values together constitute the radial spectrum of the anomalous field (Hahn *et al*, 1976). The power spectrum values is plotted versus frequency and the linear segments are located each of which the group points are due to the anomalies caused by bodies occurring within a particular depth. Kangkolo(1995) showed if Z is the mean depth of a layer, the depth factor for this ensemble of anomalies is exp (-2ZK). Thus the logarithmic plot of the radial spectrum gives a straight line whose slope is -2Z. The mean depth of burial of the ensemble is thus given by:

$$Z=\frac{m}{2} \qquad (18)$$

where, m is the slope of the best fitting straight line. Equation (18) can be applied directly if the frequency unit is in radians per kilometer. If however, the frequency unit is in cycles per kilometer, the corresponding relation can be expressed as:

$$Z=\frac{m}{4k} \qquad (19)$$

The use of the discrete Fourier transform introduces the problems of aliasing and the truncation effect (Gibb's ringing). Aliasing was minimized by digitizing the magnetic field map at unequal intervals and then interpolating at a small grid interval (Bath, 1974; Billings *et al*, 2002). The truncation effect arises in that when a limited portion of an aeromagnetic anomaly map is subjected to Fourier synthesis it becomes difficult to reconstruct the sharp edges of the anomaly with a limited number of frequencies. This truncation leads to the introduction of spurious oscillations around the region of discontinuity (it will introduce false frequencies into the spectrum). The truncation effect was reduced by applying a cosine taper to the observed data before Fourier transformation (Ofoegbu and Onuoha, 1991).

RESULTS AND DISCUSSION

Fourier transformation of magnetic anomalies data from Calabar Flank from space domain to frequency domain has provided a ready method for source depth estimation. The spectral analysis of the magnetic field over the Calabar Flank has been employed to determine the depth to intra-basement features and the thickness of the sedimentary section in the area. In this study progressively deeper sources are indicated by the lower frequency components of an anomaly. The result of this study is shown in the power spectrum plot (Fig.4). The graph is divided into linear segments corresponding to layers of magnetic sources of which two layers were evaluated and the depths of the causative layers estimated according to equation (18).

The first layer which has an average depth of 5.80km is attributed to intra-basement features (intrusives, fractures) while the second segment of the graph with a depth value of 3.11km represents the depth to basement (thickness of sedimentary section). These source depths result agreed with the result obtained from earlier gravity and magnetic survey within and adjacent area of study. This includes Umoren (1991). Using the spectral analysis approach he obtained 3.0km as the depth to basement in the Flank. The result of a gravity profile by Fairhead *et al* (1991) from the adjacent Mamfe basin shows that the central portion of the basin attained a thickness of 3km. Kangkolo (1995) obtained sediment thickness of 1km-4km in Mamfe basin. Kangkolo *et al* (1997) using the spectral analysis of aeromagnetic data showed that the thickness of sedimentary formation overlying the basement complex in the adjacent Middle Cross River Basin varied from about 1km – 4km. Nur *et al* (2003) using two dimensional spectral analysis obtained 3.33km sedimentary cover in the adjacent Benue Trough.

From the analyses of the magnetic data, optimization of digitization spacing is very important for the retrieval of true spectrum and thus the interpretational fidelity. In the frequency domain anomaly the digitization spacing should not be too close to avoid high frequency components or noise. If the spacing is too close it will eventually lead to the decrease in slope of the best fit line in fig.4 and thus the depth estimates from the power spectral analyses will relatively be on the lower side. High digitization spacing will result in higher depth estimates. Since there are three linear segments from the power spectrum (the high frequency components-noise, the intermediate and deep ensembles which generate intermediate frequencies), it is better to select the middle portion of the power spectrum for source depth determination. From the power spectrum in fig.4 the last few points have been neglected (white depth). The error in the source depth determination does not depend only on the increase of depth with source but also on the map size. Naidu (1970) recommended a minimum map size to be at least 40-50 times the source depth.

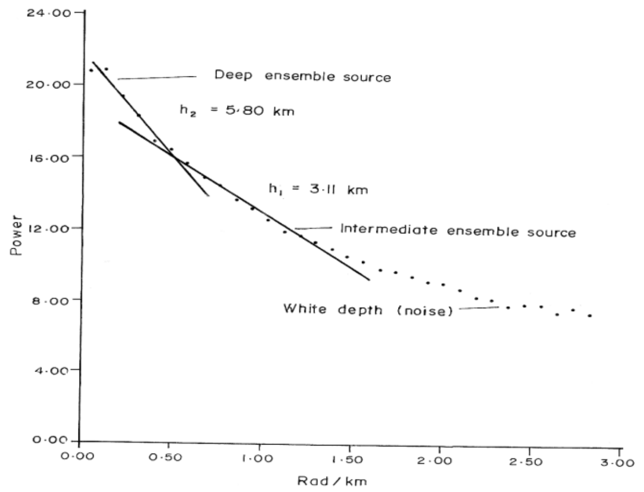

Fig.4. Power spectrum showing linear segments corresponding to depth to magnetic sources.

CONCLUSION

Power spectral analysis of the residual magnetic field over the Calabar Flank revealed two distinct layers of magnetic sources. The first layer suggests intrusions onto the basement surface and fractures within the basement (intrabasement sources) While the second layer is considered to be intrusions into the sedimentary formation. The source depths obtained from power spectral of the magnetic anomaly represents average values. The results from this study are invaluable for the initial step for the assessment of the mineral potential of the geological province.

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